Fayoum University **Applied Statistics**
Faculty of Engineering **Second Term of 2008/2009**
Dept. of Electrical Engineering (Communication and Electronics Section) **Final Exam**
Second Year **Time: 3 hours**

Model Answer

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1) $\hat{p}$ **=2/3 ,** ***n*=1600**

a) The (1- α) confidence interval for *p* is

$$\hat{p} \mu z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(1- α)=0.95 $\therefore \alpha$ **=.05** $\therefore \alpha/2$ **=0.025 and** $z_{\alpha/2} = z_{0.025} = 1.96$

The 95 % confidence interval for *p* is

$$\frac{2}{3} \mu 1.96\sqrt{\frac{1}{1600}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)} = \frac{2}{3} \mu \frac{1.96\sqrt{2}}{40*3} = \frac{2}{3} \mu 0.0230988$$

= (0.643568 , 0.689765)

b) The precision $E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\therefore$ $n = \hat{p}(1-\hat{p}) \left(\frac{z_{\alpha/2}}{E}\right)^2$

$$g(\hat{p}) = \hat{p}(1-\hat{p}) \quad \therefore g_{\max} = g(0.5) = 0.25 \quad \therefore n = \frac{1}{4}\left(\frac{z_{\alpha/2}}{E}\right)^2 = 0.25* \left(\frac{1.96}{0.02}\right)^2 = \mathbf{2401}$$

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2) The subscript 1 denotes cotton fiber

The subscript 2 denotes acetate

$n_1 = 25$ **,** $s_1 = 1.5$ **,** $\bar{x}_1 = 20$

$n_2 = 25$, $s_2 = 1.25$, $\bar{x}_2 = 12$

$y \equiv x_1 - x_2$, $Y \equiv X_1 - X_2$, $\bar{Y} \equiv \bar{X}_1 - \bar{X}_2$

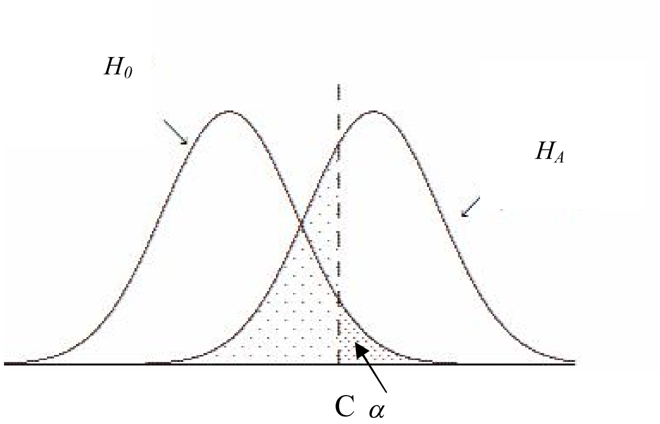
$E(\bar{Y}) = E[\bar{X}_1] - E[\bar{X}_2] = E[X_1] - E[X_2] = \mu_1 - \mu_2$

$\sigma_{\bar{Y}}^2 = \text{var}(\bar{Y}) = \text{var}(\bar{X}_1 - \bar{X}_2) = \text{var}(\bar{X}_1) + \text{var}(\bar{X}_2)$ **=** $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$ **=** $\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

$s_{\bar{Y}} = s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ **where** $s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}$ $\Rightarrow$ $s^2 = \frac{(24)(1.5)^2 + (24)(1.25)^2}{48}$

$s^2 = \frac{1}{2}\left[(1.5)^2 + 1.25)^2\right]$ **=1.90625=** $(1.380670)^2$

$s_{\bar{Y}} = 1.380670\sqrt{\frac{1}{25} + \frac{1}{25}}$ **= 0.390512**



$H_0: \quad \mu_y = 0$

$H_A: \quad \mu_y > 0$

$\frac{C - 0}{s_{\bar{Y}}} = \quad t_{n_1 + n_2 - 2, \alpha}$ $\quad C = s_{\bar{Y}}\ t_{n_1 + n_2 - 2, \alpha}$

$t_{n_1 + n_2 - 2, \alpha} = t_{48,\ 0.05} = z_{0.05} = 1.645$

$\text{C} = (0.390512) * (1.645) = .642393$

$\bar{y} \equiv \bar{x}_1 - \bar{x}_2$ **=20-12=8** $\Rightarrow$ $\bar{y} > C$

Decision : We reject H_0 $\Rightarrow$ **We accept** H_A

There is strong evidence that $\mu_1 > \mu_2$

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3) ***n*****=5**

a) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 3$, $s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2 = 0.815 = (0.9027735)^2$

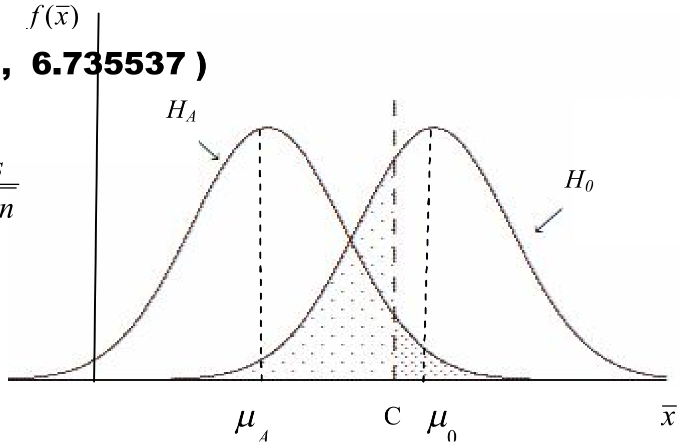
b) The (1- α) confidence interval for σ^2 is $\left(\frac{(n-1)s^2}{\chi^2_{n-1,\ \alpha/2}} \quad , \quad \frac{(n-1)s^2}{\chi^2_{n-1,\ 1-\alpha/2}} \right)$

(1- α)=0.95 $\Rightarrow$ α **=.05** $\Rightarrow$ $\alpha/2$**=0.025**

$\chi^2_{n-1,\ \alpha/2}$ **=** $\chi^2_{4,\ 0.025}$ **=11.143** **and** $\chi^2_{n-1,\ 1-\alpha/2}$ **=** $\chi^2_{4,\ 0.975}$ **=0.484**

The 95 % confidence interval for σ^2 is

$\left(\frac{4*0.815}{11.143} \quad , \quad \frac{4*0.815}{0.484} \right)$ **= (0.292560 , 6.735537)**



c) $\frac{C-\mu_0}{s/\sqrt{n}} = -t_{n-1,\alpha} \Rightarrow \quad C = \mu_0 - t_{n-1,\alpha} \frac{s}{\sqrt{n}}$

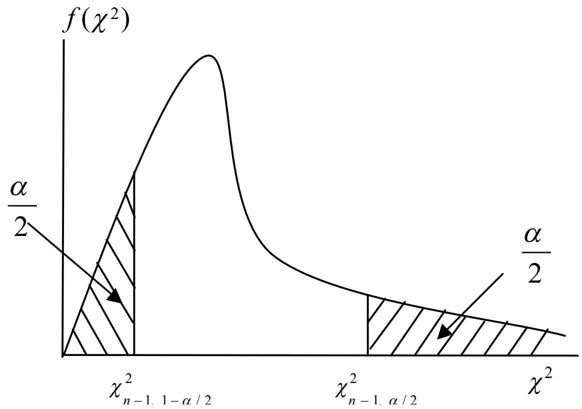
$t_{n-1,\alpha}$ **=** $t_{4,0.1}$ **=1.533** $\Rightarrow$

$C = 3.5 - 1.533 \frac{\sqrt{0.815}}{\sqrt{5}}$ **= 2.881**

$\bar{x} = 3 \quad \Rightarrow \quad \bar{x} > C$

Decision : We accept H_0 **.**

d) The decision criteria :



C_1= $\frac{\sigma_0^2\ \chi^2_{n-1,\ 1-\alpha/2}}{n-1}$ **and** **C_2=** $\frac{\sigma_0^2\ \chi^2_{n-1,\ \alpha/2}}{n-1}$

α **=.05** $\Rightarrow$ $\alpha/2$**=0.025**

$\chi^2_{n-1,\ \alpha/2}$ **=** $\chi^2_{4,\ 0.025}$ **=11.143**

$\chi^2_{n-1,\ 1-\alpha/2}$ **=** $\chi^2_{4,\ 0.975}$ **=0.484**

C_1= $\frac{(1)*(0.484)}{4}$ **= 0.121 ,** **C_2=** $\frac{(1)*(11.143)}{4}$ **= 2.78575**

$s^2 = 0.815 \quad \Rightarrow \quad C_1 < s^2 < C_2 \quad \Rightarrow$ **Decision : We accept** H_0**.**

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4) A_i**: The event that engine *i* is available when needed.**

$P(A_i) = 0.96$ ***i* =1, 2**

A_1 **and** A_2 **are statistically independent.**

a) $P(\overline{A}_1 \cap \overline{A}_2) = P(\overline{A}_1) * P(\overline{A}_2) = (1-0.96)^2 = (0.04)^2 = 0.0016$

b) $P_b = P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1) * P(A_2)$

$= 0.96 + 0.96 - (0.96)^2 = 0.9984$

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5) Success: getting a head in one toss

***p*=0.45** $\Rightarrow$ ***q*=1-*p*=1-0.45=0.55**

***X* : number of failures before the r th success** $\Rightarrow$ ***X* follows the negative binomial distribution**

$p(x) = {}_{x+r-1}C_x \; p^r \; q^x \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots.$

***r* = 4, *x*+*r* = 11** $\Rightarrow$ ***x* = 11-4 = 7**

$p(7) = {}_{10}C_7\,(0.45)^4\,(0.55)^7$ $p(7) = 0.0749152$

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6)

x	**1**	**2**	**3**	**4**	**5**	**6**
p(x)	**1/6**	**1/6**	**1/6**	**1/6**	**1/6**	**1/6**

$$\mu_x = E[X] = \sum_{x=1}^{6} x\ p(x) = \frac{1}{6}\sum_{x=1}^{6} x \ = \frac{21}{6} = 3.5$$

$$E[X^2] = \sum_{x=1}^{6} x^2\ p(x) = \frac{1}{6}\sum_{x=1}^{6} x^2 \ = \frac{91}{6} = 15.166667$$

$$\sigma_X^2 = E(X^2) - \mu_X^2 = 2.916667$$

***X* and *Y* are identically distributed**

$$Z = X + 3Y - 5$$

$$\mu_z = E[Z] = E[X] + 3E[Y] - 5$$

$$Z - \mu_z = (X - E[X]) + 3(Y - E[Y])$$

$$(Z - \mu_z)^2 = (X - E[X])^2 + 9(Y - E[Y])^2 + 6(X - E[X])(Y - E[X])$$

$$E\left[(Z - \mu_z)^2\right] = E\left[(X - E[X])^2\right] + 9E\left[(Y - E[Y])^2\right] + 6E[(X - E[X])(Y - E[Y])]$$

$$\sigma_Z^2 = \sigma_X^2 + 9\sigma_Y^2 + 6\,\mathrm{cov}(X,Y)$$

$\mathrm{cov}(X,Y)$ **=0** $\quad \Rightarrow \quad \sigma_Z^2 = (1+9)\sigma_X^2 = 10\sigma_X^2 = 29.16667$

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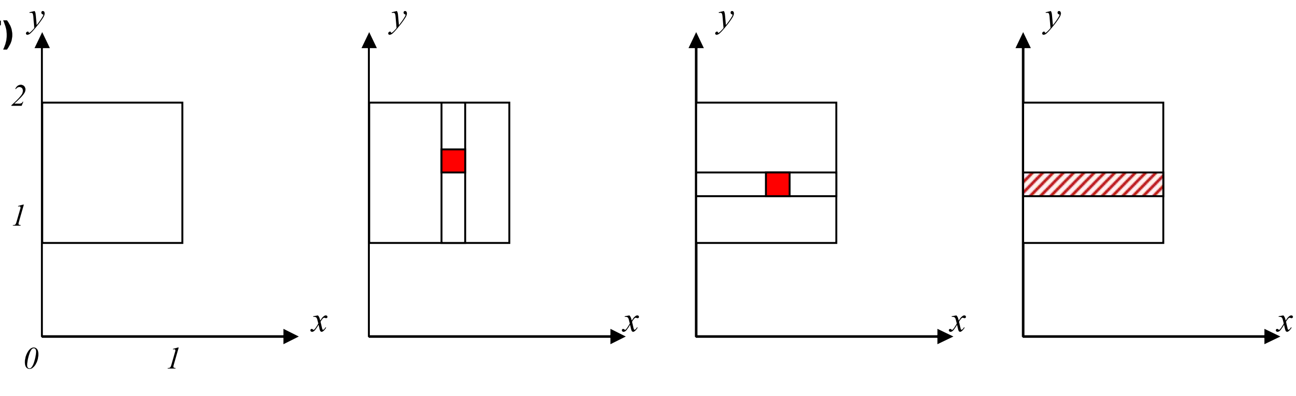
7)



(a) (b) (d)

a) $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$

For x < 0 or x > 1 : $f_X(x)$ **= 0**

For 0 < x < 1 :

$$f_X(x)=\int_1^2 \frac{2}{7}(x+2y)\,dy=\frac{2}{7}(\frac{1}{2})(\frac{1}{2})(x+2y)^2\Big|_{y=1}^{2}$$

$$=\frac{1}{14}\left[(x+4)^2-(x+2)^2\right]=\frac{1}{14}\left[(8x+16)-(4x+4)\right]=\frac{1}{14}(4x+12)=\frac{2}{7}(2x+3)$$

Therefore

$$f_X(x)=\begin{cases}\frac{2}{7}(x+3) & 0<x<1\\ 0 & \text{otherwise}\end{cases}$$

b) $f_Y(y)=\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

For y < 1 or y > 2 : $f_Y(y)$ **= 0**

For 1 < y < 2 :

$$f_Y(y)=\int_0^1 \frac{2}{7}(x+2y)\,dx=\frac{2}{7}(\frac{1}{2})(x+2y)^2\Big|_{x=0}^{1}=\frac{1}{7}\left[(1+2y)^2-(2y)^2\right]=\frac{1}{7}(1+4y)$$

Therefore

$$f_Y(y)=\begin{cases}\frac{1}{7}(4y+1) & 1<y<2\\ 0 & \text{otherwise}\end{cases}$$

c) $\mu_X=E[X]=\int_{-\infty}^{\infty} xf_X(x)\,dx=\int_0^1 \frac{2}{7}(x^2+3x)\,dx=\frac{2}{7}(\frac{x^3}{3}+\frac{3}{2}x^2)\Big|_0^1=\frac{2}{7}(\frac{1}{3}+\frac{3}{2})=\frac{2}{7}\left(\frac{2+9}{6}\right)$

$$=\frac{11}{21}=0.523809523$$

$$\mu_Y=E[Y]=\int_{-\infty}^{\infty} yf_Y(y)\,dy=\int_1^2 \frac{1}{7}(4y^2+y)\,dy=\frac{1}{7}(\frac{4}{3}y^3+\frac{y^2}{2})\Big|_1^2=\frac{1}{7}\left[(\frac{32}{3}+2)-(\frac{4}{3}+\frac{1}{2})\right]$$

$$=\frac{1}{7}\left[\frac{38}{3}-\frac{11}{6})\right]=\frac{1}{7}\left(\frac{65}{6}\right)=\frac{65}{42}=1.547619$$

d) $E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy \; f_{X,Y}(x,y)dxdy = \int_{y=1}^{2}\int_{x=0}^{1} \frac{2}{7} y(x^2+2xy)\,dxdy$

$= \int_{y=1}^{2} \frac{2}{7} y(\frac{x^3}{3}+x^2 y)\Big|_{x=0}^{1} dy = \int_{1}^{2} \frac{2}{7} y(\frac{1}{3}+y)dy = \int_{y=1}^{2} \frac{2}{7}(\frac{y}{3}+y^2)dy$

$= \frac{2}{7}(\frac{y^2}{6}+\frac{y3}{3})\Big|_{1}^{2} = \frac{2}{7}\left[(\frac{2}{3}+\frac{8}{3})-(\frac{1}{6}+\frac{1}{3})\right] = \frac{2}{7}\left(3-\frac{1}{6}\right) = \frac{2}{7}\left(\frac{17}{6}\right) = \frac{17}{21} = 0.809524$

$\mathrm{cov}(X,Y) = E[XY] - \mu_X \mu_y = \frac{17}{21} - \left(\frac{11}{21}\right)\left(\frac{65}{42}\right) = -0.00113379 = -\frac{1}{882}$

e) $\mathrm{cov}(X,Y) \neq 0 \;\Rightarrow$ **X and Y are not uncorrelated** $\Rightarrow$ **X and Y are not statistically independent.**

f) $E[g(XY)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(xy) \; f_{X,Y}(x,y)dxdy = \int_{y=1}^{2}\int_{x=0}^{1} (\frac{x}{y^3}+x^2 y)\left(\frac{2}{7}\right)(x+2y)\,dxdy$

$= \frac{2}{7}\int_{y=1}^{2}\int_{x=0}^{1}\left(x^3 y + \frac{x^2}{y^3} + 2x^2 y^2 + 2\frac{x}{y^2}\right) dxdy = \frac{2}{7}\int_{y=1}^{2}\left(\frac{x^4}{4}y + \frac{x^3}{3y^3} + \frac{2}{3}x^3 y^2 + \frac{x^2}{y^2}\right)\Bigg|_{x=0}^{1} dy$

$= \frac{2}{7}\int_{y=1}^{2}\left(\frac{1}{4}y + \frac{1}{3y^3} + \frac{2}{3}y^2 + \frac{1}{y^2}\right)dy = \frac{2}{7}\left(\frac{1}{8}y^2 - \frac{1}{6y^2} + \frac{2}{9}y3 - \frac{1}{y}\right)\Bigg|_{y=1}^{2}$

$= \frac{2}{7}\left[\left(\frac{1}{2}-\frac{1}{24}+\frac{16}{9}-\frac{1}{2}\right)-\left(\frac{1}{8}-\frac{1}{6}+\frac{2}{9}-1\right)\right] = \frac{2}{7}\left(\frac{-3+128-9+12-16+72}{72}\right)$

$= \frac{2}{7}\left(\frac{184}{72}\right) = \frac{46}{63} = 0.730159$